12 Group actions

12.1. Consider action of the group $(\mathbb{R}, +, -, 0)$ on the plane $\mathbb{R}^2$ such that for any $u \in \mathbb{R}$

(a) $(a, b) \to (a + u, b)$

(b) The plane is rotated by u degrees counterclockwise.

Prove that this is a group action and for $(a, b) \in \mathbb{R}$ describe its orbit and stabiliser.

12.2. Calculate, how many different bracelets can be made with six red and three white beads, using all nine beads.

12.3. Suppose you have 8 red and 8 blue equilateral triangles. Count the number of ways one can build an equilateral triangle with edges of quadruple sizes

(a) up to rotations,

(b) up to rotations and reflections.

12.4. Consider the action of the group $G = \S_n$ on the set $X = \{(a,\ b) : 1 \leq a, b \leq n\}$, with the permutation π acting on the components, i.e. $\pi((a, b)) = (\pi(a), \pi(b))$. Determine

(a) $|X/\sim|$, the number of cosets of the equivalence $\sim$,

(b) the number of elements of $[(1,1)]_\sim$ and $[(1,2)]_\sim$,

(c) indexes $[G : G_{(1,1)}]$ and $[G : G_{(1,2)}]$.

12.5. Determine how many ways the faces of the regular tetrahedron can be coloured by n colours up to rotations.